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Modelling the wave energy spectrum in shallow water

by

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Introduction

Wave growth is restricted, and the wave energy spectrum changes from the deep water shape, when the wavelength is longer than approximately three times the water depth. There is an increasing demand for high quality forecast wave spectra in shallow water, particularly for complex heavy lifting operations in water less than 200m depth. Even a very low amplitude swell, if of the resonant period of the lifting barge, may hold up operations for several days. The performance of numerical wave prediction models in shallow water is being pushed to the limit in such cases. Because the long wavelength swell energy is crucial to the operation it is important that the wave model representation of the spectrum in shallow water is correct, since the swell energy may be affected by the sea bed even though locally generated windsea may not be. Figure 1 shows the approximate depth at which deep water waves of a given frequency first feel the sea bed. The UKMO wave model accounts for depth-dependent group velocity, refraction of wave energy and energy dissipation by bottom friction. The formulation of the shallow water terms has remained unchanged since the SWIM intercomparison of models in 1985 (SWIM group, 1985).

The aim of this Technical Note is to review progress in modelling shallow water wave spectra over the last ten years, and if necessary to recommend revisions to the formulation of the UKMO wave model in the light of recent developments. Section 1 reviews the physics of wave energy dissipation in shallow water; Section 2 considers the impact of enhanced wave-wave interactions in shallow water and compares model spectra with the TMA family of shallow water spectra. A summary and recommendations are in Sections 3 and 4.

1 Energy dissipation in shallow water.

The UKMO wave model currently uses a parametrisation for bottom friction due to Collins (1972). A similar expression derived from the JONSWAP results (Bouws and Komen 1983) is used in the 3rd generation WAM model run at ECMWF.

The UKMO model expression (used in the SWIM 1985 intercomparison) is:

$$S_{bf} = -C\, g\, k^2\, \{U\}\, E(f,\theta) / [\, 2\pi f \cosh(kH)]^2 \qquad (1a)$$

where k is the shallow water wave number, H is the depth, g the acceleration due to gravity, E the wave energy density and {U} is the mean orbital velocity at the sea bed of the water in

the wave, given by:

$$\{U\}^2 = \int\int [gk/2\pi f \cosh(kH)]^2 \, E \, df \, d\theta \qquad (1b)$$

Note that Eqn (1a) is different from the expression originally proposed by Golding (1983), which was:

$$S_{bf} = -C \, g \, k \, (c_g/2\pi) \, \{U\} \, E(f,\theta) \, / \, [\, 2\pi f \cosh(kH)]^2 \qquad (2)$$

where c_g is the group velocity.

The WAM model uses:

$$S_{bf} = -C_{WAM} \, k \, E(f,\theta) \, / \, \sinh(2kH) \qquad (3)$$

which is the same in form as (1a) when $2C_{UK}\{U\} = C_{WAM}$. The UKMO wave model uses a value 0.005 for the constant C, chosen by Golding (1983) to provide approximate agreement with empirical expressions for wave generation in shallow water, although for the SWIM study cases a value C = 0.015 was used. The WAM model uses a value 2x0.038/g. Equating the two expressions, the bottom friction in the WAM model is identical to the UKMO formulation if $\{U\} = 0.77$m/s. This is a large value, as typical wave orbital velocities at the sea bed will be of the order of centimetres per second. The WAM model may apply more bottom friction than the present UKMO model.

There are several different physical processes that act to dissipate wave energy at the sea bed. Which process is dominant at a given site depends upon the nature of the sea bed. The main processes, described in the review by Shemdin et al (1978), are percolation, bottom friction, wave-bottom interaction and bottom scattering due to ripples on the sea bed. In addition, the enhanced nonlinear interactions in shallow water may act to transfer relatively more wave energy than in deep water to lower frequencies, and at these lower frequencies and longer wavelengths the wave energy will be dissipated through interaction with the sea bed. The expression (1a) was derived by Collins (1972) assuming a quadratic friction law for the drag in the sea-bed boundary layer, and the result theoretically applies for both bottom friction and the effects of percolation, with a different coefficient for each process (Shemdin et al, 1978).

Percolation occurs when the bottom is composed of coarse sand grains, and is due to the water flow within the sand layer. Bottom friction accounts for the drag exerted on the wave

by the turbulent bottom boundary layer, and particularly depends on the roughness of the sea bed. Bottom friction is important where the sea bed is of fine sand. In storms the sand at the sea bed is formed into ripples, and the larger these are, the more drag is exerted. Ripples are also formed by tidal currents, or may exist from a previous storm event. Bottom friction may vary by an order of magnitude depending on ripple height and spacing, and Weber (1988) has suggested that the bottom friction coefficient C should be scaled with the seabed U* and water depth.

If the sea bottom is composed of soft mud, then the wave flow may induce viscous or elastic flow in the mud, thus altering the wave flow and dissipating wave energy. This process is mainly found close to estuaries of large rivers, where silt and mud may be deposited on the sea bed, and is not accounted for in wave models at present. It is unlikely to be important at the resolution of current operational global and regional wave models.

Weber (1991) proposed a form for the coefficient C in Eqn (3) dependent upon the sea bottom friction velocity, roughness and the wave frequency. This was based on an eddy viscosity model of the sea bed boundary layer flow. A simplified expression was presented and compared with an expression derived from a quadratic drag law. It was found that the simplified expression from the eddy viscosity model gave the best results in two test hindcasts run with the WAM model, consisting of a severe storm and a swell episode in the southern North Sea.

Weber took an average bottom roughness length corresponding to a ripple height of 2cm. It was shown that this would be appropriate for the North Sea in the conditions of the hindcast. The simplified expression derived by Weber to replace C_{WAM} is:

$$C_{WEBER} = \{U\} \exp(-8.34 + 6.34\, Z_b^{0.08}) \qquad (4)$$

In the hindcasts Weber proposed a value $Z_b = 0.04$m. Taking this value we find that $C_{WEBER} = 0.032\ \{U\}$, so the expression proposed by Weber is identical to that used by the UKMO wave model, to within a constant. For the roughness value suggested by Weber the corresponding coefficient for the UKMO wave model is C=0.016 which is three times larger than the current value of 0.005, and close to the value used in the SWIM study. The present operational UKMO wave model apparently underestimates wave energy dissipation by bottom friction.

The bottom friction coefficient derived from the eddy viscosity model may be extended to allow for the effects of a current on the sea bed boundary layer. Weber found that including the tidal current increased bottom friction by at most 5% in a severe storm hindcast, which was considered negligible, and by at most 10% for an extreme swell event, which was considered just acceptable. Weber suggested however that for isolated, less extreme swell the tidal effect on bottom friction should not be neglected. There was a significant increase in dissipation of the current energy.

2 Wave-wave nonlinear interactions.

2.1 comparison of model spectra with empirical spectra.

In shallow water the rate of transfer of wave energy to lower frequencies by nonlinear wave-wave interactions is enhanced relative to the deep water values, by up to an order of magnitude (Herterich and Hasselmann, 1980). In deep water the combined action of the wind input, dissipation and nonlinear transfer leads to a self similar family of spectra - the JONSWAP spectral family (JONSWAP 1973). The same ideas were extended to shallow water by Bouws et al (1985 and 1986) using three datasets of spectral measurements. They defined the so-called TMA spectral family.

The self similarity of growing windsea spectra in shallow water was demonstrated as an extension of the deep-water JONSWAP family of spectra. The TMA spectral form, based on measurements made at Texel, and in the MARSEN and ARSLOE experiments (hence TMA), multiplies the energy level in each frequency component of a JONSWAP spectrum by a frequency and depth dependent factor. The coefficient ϕ is a function of ω_H, where $\omega_H = \omega$ sqrt(H/g) (with H the water depth and g gravity). A graph of ϕ is shown in Figure 2. In deep water ϕ equals one, so the TMA spectrum is the same as the JONSWAP spectrum; in shallow water $0 < \phi < 1$, so the energy levels are reduced in those frequencies.

Comparison of model spectra with a calculated TMA spectrum revealed several points. The TMA spectrum implicitly accounts for both dissipation of wave energy by bottom friction, and for the enhanced nonlinear transfer in 'shallow' water. Thus a comparison of TMA wave height with the wave height predicted by the UKMO shallow water wave model for a fully developed spectrum, will be a guide to the correctness of the the amount of bottom dissipation included in the UKMO model. Also a comparison of the 1D frequency spectra revealed the importance of the enhanced nonlinear transfer in shallow water, which is not accounted for in the UKMO wave model.

In the UKMO wave model wave energy is removed from the spectrum by a term accounting for bottom friction. This means that in shallow water the model equilibrium energy is less than the deep water value Epm. This difference influences the spectral shape because the spectral lookup table parameters are calculated according to the deep water values for the Pierson-Moskowitz spectrum:

a) $$\gamma = 1 + 2.3*[\ 1-(EWS/EPM)^2\ ]$$

b) $$F_j = F_{PM}\ (EWS/EPM)^{-0.33} \qquad (5)$$

where EWS is the model windsea energy, and EPM is the Pierson-Moskowitz energy for the windspeed. [γ is the JONSWAP spectral peak enhancement parameter. For a fully developed spectrum $\gamma=1$. The mean value found during JONSWAP was $\gamma=3.3$]. Thus from (5a) we may expect in shallow water that γ will be much larger than 1, even when the shallow water spectrum is fully developed, and from (5b) that F_j will be greater than F_{PM}. So compared to the TMA spectrum the UKMO wave model equilibrium shallow water spectrum will be more peaked (γ too large) and at a higher peak frequency. Figure 3 shows the model and TMA spectra for a windspeed of 15m/s and a depth of 200m, which is deep water. The spectra are similar, but not identical, because of differences in the method of calculation of spectral parameters (Details of the calculation of the TMA spectrum are in Appendix A). The wave heights are the same. Figure 4 shows model and TMA spectra for depths of 50m, 25m and 20m for a windspeed of 15m/s. Model spectra are shown for both the current value Cbf=0.005, and for the value Cbf=0.016 suggested by Section 1. The wave heights correspond in deep water, but moving into shallow water it is clear that the TMA equilibrium wave height is lower than that predicted by the present UKMO model for this windspeed. Also the peak frequency of the TMA spectrum is at a lower frequency than the model peak frequency, and the TMA spectrum is much less peaked than the model spectrum.

Figure 5 shows, for a windspeed of 15m/s, the variation with depth of wave height (Figure 5a), and peak frequency and peak period (Figure 5b), for both the model and TMA spectra, using values of Cbf=0.005 and 0.016 in the model. With Cbf=0.016 the model wave heights agree closely with the TMA heights as the depth decreases. With Cbf=0.005 the model heights are higher than TMA heights, suggesting an underestimate of the energy dissipation from bottom friction. However the reduction in wave energy causes the model spectrum to be more peaked, and the peak frequency to be higher for Cbf=0.016 (Figure 5b).

Figure 6 shows the variation of waveheight with depth (as Figure 5a) for a windspeed of 10m/s. Here, although the calculated TMA energy is slightly greater than model values, with Cbf=0.016 the model spectrum is first noticeably affected in a depth of 50m, as is the TMA, whereas with Cbf 0.005 there is little impact until the water is shallower than 50m depth. Figure 7 shows the variation of wave height with windspeed for a depth of 20m. Again the model values with Cbf 0.016 are closer to TMA heights, and are lower than the model values using Cbf 0.005.

2.2 Comparison of empirical spectra with a 3rd generation wave model

The author has found only one publication in which the shallow water nonlinear transfer has been explicitly computed. Weber (1988) modified the Exact-NL model of Hasselmann et al to represent correctly the nonlinear transfer in shallow water, and to account for energy dissipation by bottom friction. The expression for bottom friction was that used in the WAM model, described in Section 1. Weber compared the shallow water Exact-NL model results with results from the SWIM intercomparison of the UKMO wave model, the Dutch GONO and German HYPAS models (SWIM, 1985). The second generation UKMO and GONO wave models used a similar formulation for bottom friction, with a standard 2G approach to nonlinear transfer; the HYPAS model produced a spectrum based on the TMA spectrum.

The SWIM intercomparison had shown that, as found above in the comparison of model and TMA spectra, the UKMO and GONO model spectra had a higher peak frequency than the equilibrium TMA spectrum. Weber found that the shallow water Exact-NL model gave asymptotic results for both peak frequency and total wave energy that were closer to the UKMO and GONO models than to the TMA HYPAS model. Some doubts had been expressed in the SWIM report (SWIM 1985) about the homogeneity of the datasets used to derive the TMA spectra. It was felt that energy advected from outside the region may have been present in some of the observed spectra. This would have resulted in a lower mean frequency for the spectrum.

Although the comparison of UKMO model and TMA spectra described above revealed significant differences in peak frequency and period, particularly for depths less than 50m, it is not yet clear that the TMA spectrum is universally applicable. The difference between the model results and the TMA spectral form based on observations has not yet been explained in the literature, and at least one research group (Piroth, 1993, personal communication at ASI symposium, Marseille) has used the TMA spectrum to transform an existing deep water

spectrum that is moving into shallow water. This is quite different from representing the spectrum of waves developing wholly within shallow water.

2.3 Wave steepness and whitecapping

Weber (1988) also examined the possible effect of increased wave steepness Ek^2 on dissipation due to whitecapping. She found that the mean wave steepness did not vary significantly with water depth, so dissipation due to whitecapping was also approximately constant over different water depths.

3 Conclusions

The formulation of bottom friction in the UKMO model is sound, and accounts for the important physical processes present in the model area. Wave induced bottom motion is not included, but this is only important in areas such as river estuaries where mud or silt is deposited. These areas are not resolved by present models.

There is some evidence that the spectral peak frequency is overestimated by the UKMO model for moderate or strong winds in shallow waters (<50m). The behaviour of observed spectral peak frequency in shallow water remains unknown and the differences between the TMA spectra, and UKMO and EXACT-NL model results remain unexplained.

The form of bottom friction coefficient used, C{U}, is in line with the findings of Weber (1991) based on an eddy viscosity model for the sea bottom boundary layer for a given roughness length, however the value of the coefficient C used at present is too low. It is possible that the coefficient calculated by Golding (1983) was not changed when the formulation was changed from Eqn(2) to Eqn (1a).

4 Recommendations

a) We should change the bottom friction coefficient used in the wave model to 0.016, in line with the results of Weber 1991. This corresponds to a simplified expression derived from an eddy-viscosity model of the sea bottom boundary layer for a roughness length of 0.04m.

b) It is probably not worth extending the coefficient C to depend on bottom roughness length for the present resolution of the wave model, although the possibility should be noted for any future higher resolution work. (Eqn 4)

c) Further work is required to settle the question of the spectral peak frequency behaviour for fully developed spectra in shallow water. Is the asymptotic peak frequency for a given windspeed higher in shallow water ?

d) Because of the uncertainty of what is represented by the TMA spectrum, it is not yet appropriate to convert the UKMO wave model to use this spectral form.

APPENDIX A: Calculation of the TMA spectrum

1) Assuming a fully developed spectrum we have a value 0.0081 for the Phillips constant α. (Hasselmann et al 1976, following Eqn 2.1)

2) From Bouws et al (1985) Eqn 14, the expression $k_m^2 = 0.25\ \alpha / E$ defines the wavenumber k_m corresponding to the peak frequency f_m for the TMA spectrum at this depth. The value Epm is used for the total wave energy E in this expression.

3) From k_m calculate f_{TMA} using the shallow water dispersion relation:

$$(2\pi f_{TMA})^2 = g\, k_m \tanh(k_m H)$$

f_{TMA} is the peak frequency of the required spectrum, and defines the entry to the wave model lookup table, IFJ. For a fully developed spectrum a value of $\gamma = 1$ is used.
Each frequency component of the JONSWAP spectrum from the wave model lookup table is then multiplied by the factor $\phi(\omega_H)$.

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Weber, Nanne (S L)	1991	Bottom friction for wind sea and swell in extreme depth-limited situations. **JPO 21** pp 149-172

FIGURES

Figure 1 — The depth at which deep water waves of a given frequency first feel the sea bed. (Depth plotted is one quarter the deep water wavelength).

Figure 2 — The function PHI used by the TMA spectrum to modify the energy of the JONSWAP spectrum in each frequency component.

Figure 3 — Model and TMA spectra for deep water. U=15m/s.

Figure 4 — Model and TMA spectra for U=15m/s, and for Cbf=0.005 and 0.016.
a) depth 50m
b) depth 25m
c) depth 20m

Figure 5 — Variation of model and TMA spectral parameters with depth for U=15m/s.
a) wave height
b) peak frequency Fj, and peak period 1/Fj

Figure 6 — Variation of wave height with depth for U=10m/s.

Figure 7 — Variation of wave height with windspeed for a depth of 20m.

Figure 1

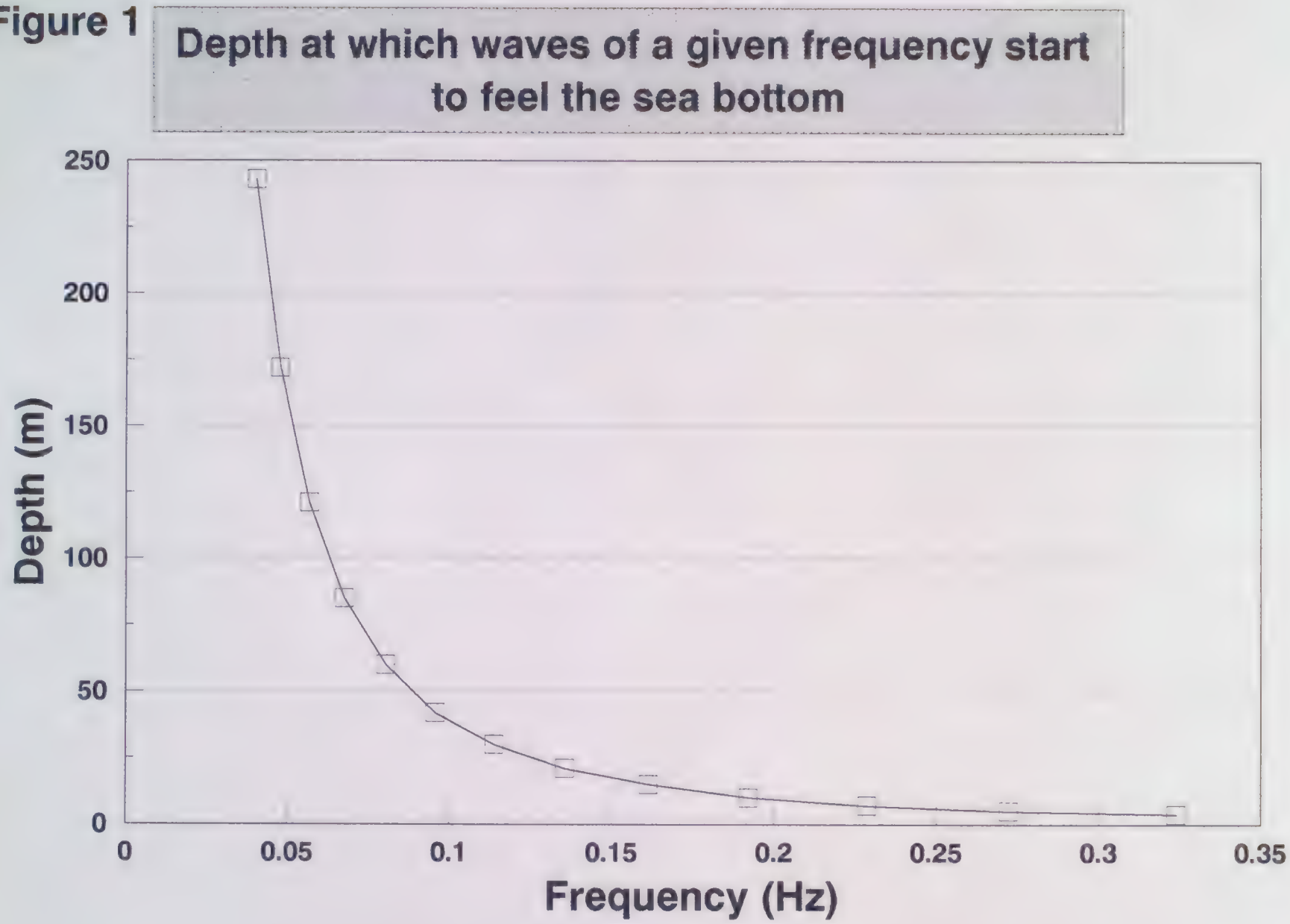

Depth at which waves of a given frequency start to feel the sea bottom
Depth (m)
250
200
150
100
50
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Frequency (Hz)

Figure 2

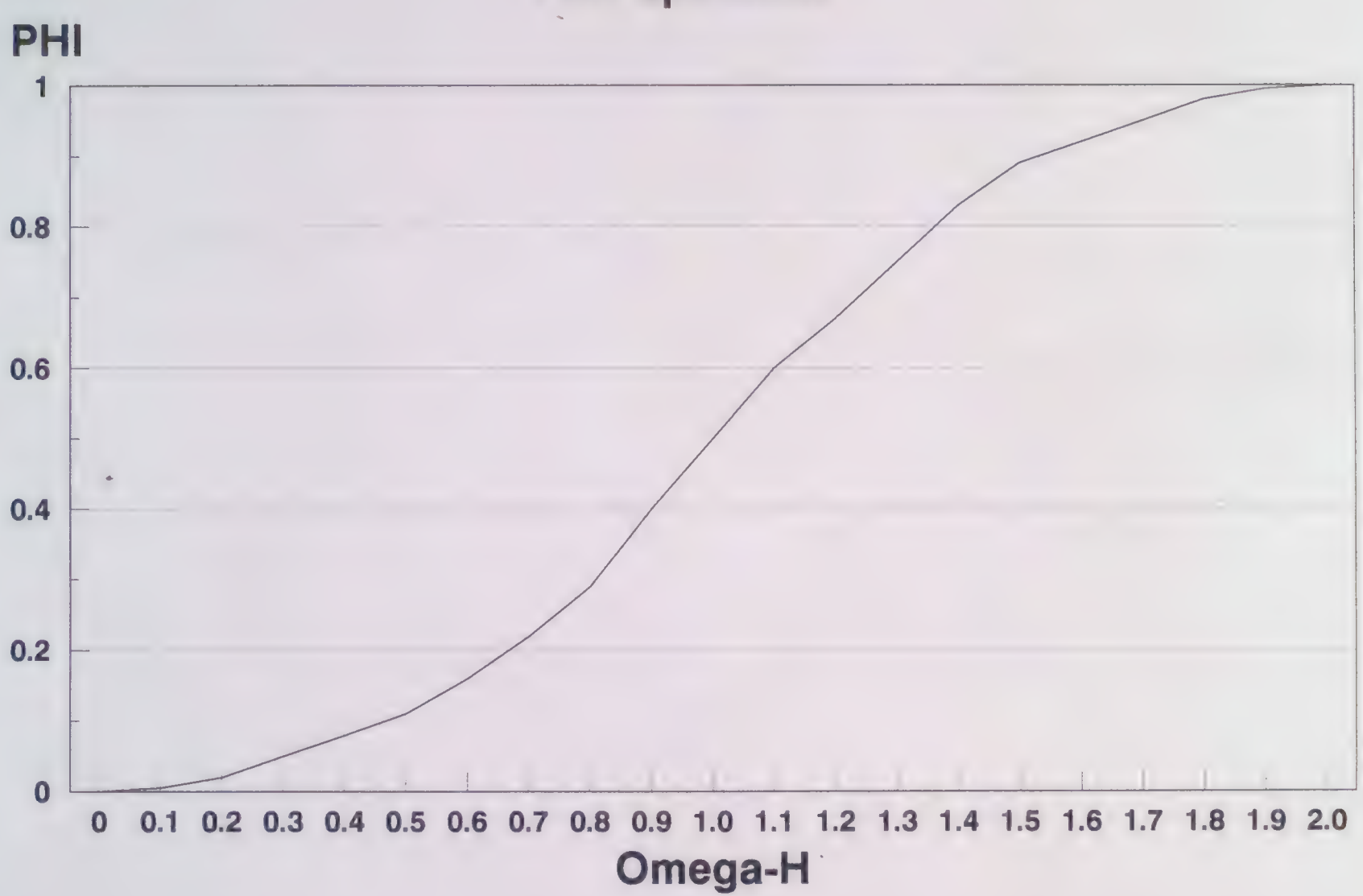

PHI(Wh) Function
TMA spectrum
PHI
1
0.8
0.6
0.4
0.2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
Omega-H

Figure 3 Shallow water spectra
Depth 200m U=15m/s

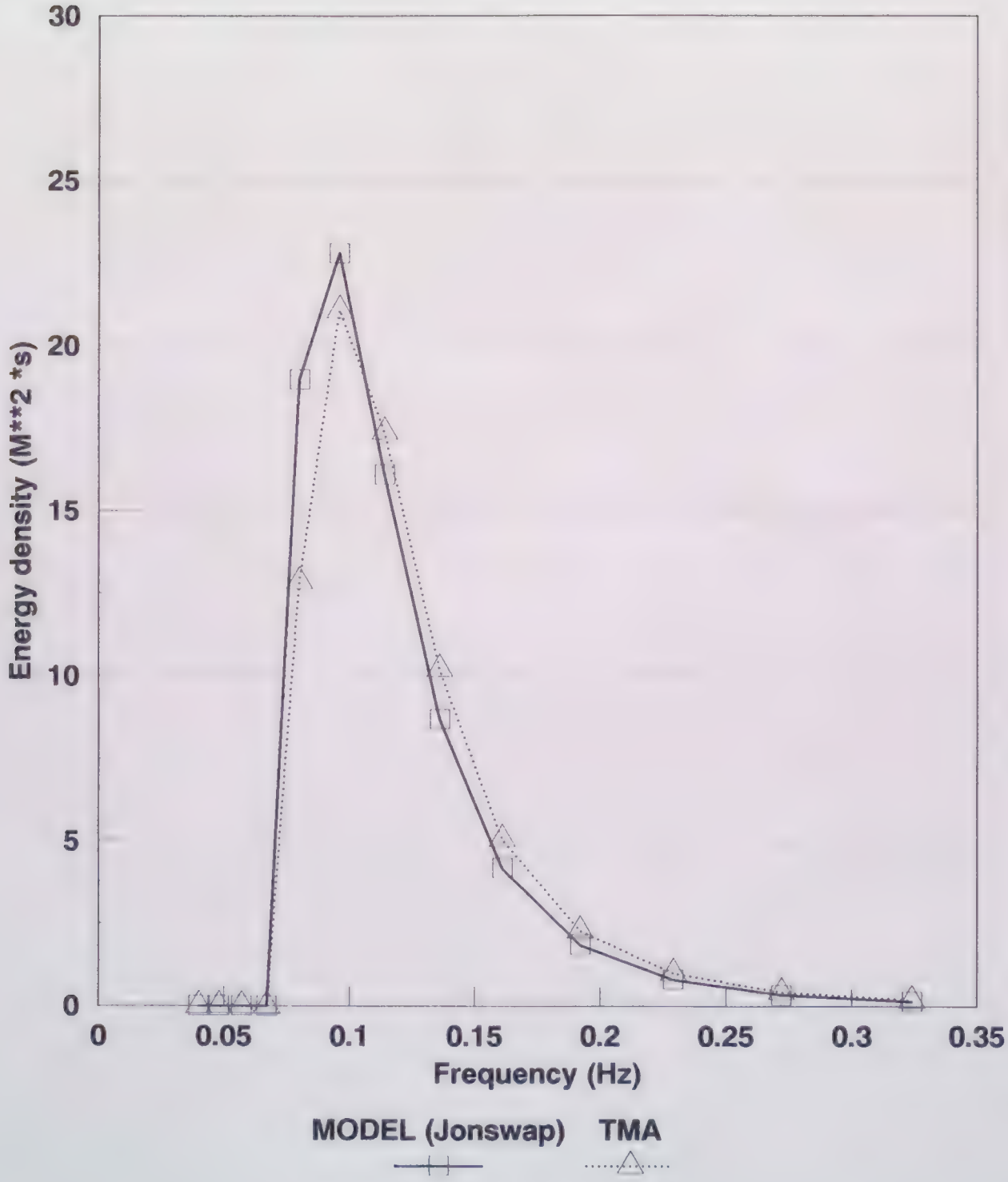


Figure 4a Shallow water spectra
DEPTH 50m U=15m/s

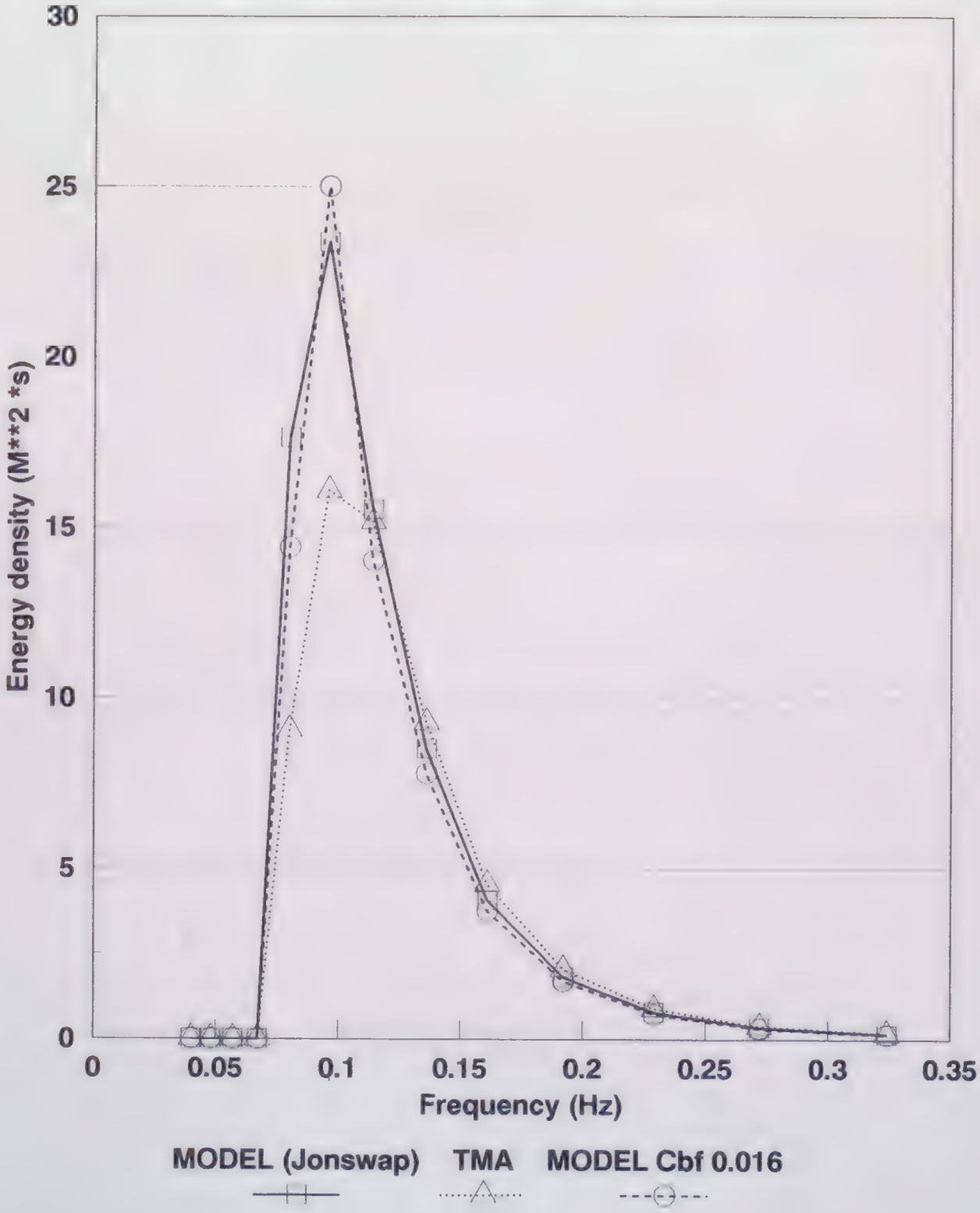


Figure 4b Shallow water spectra
DEPTH 25m U=15m/s

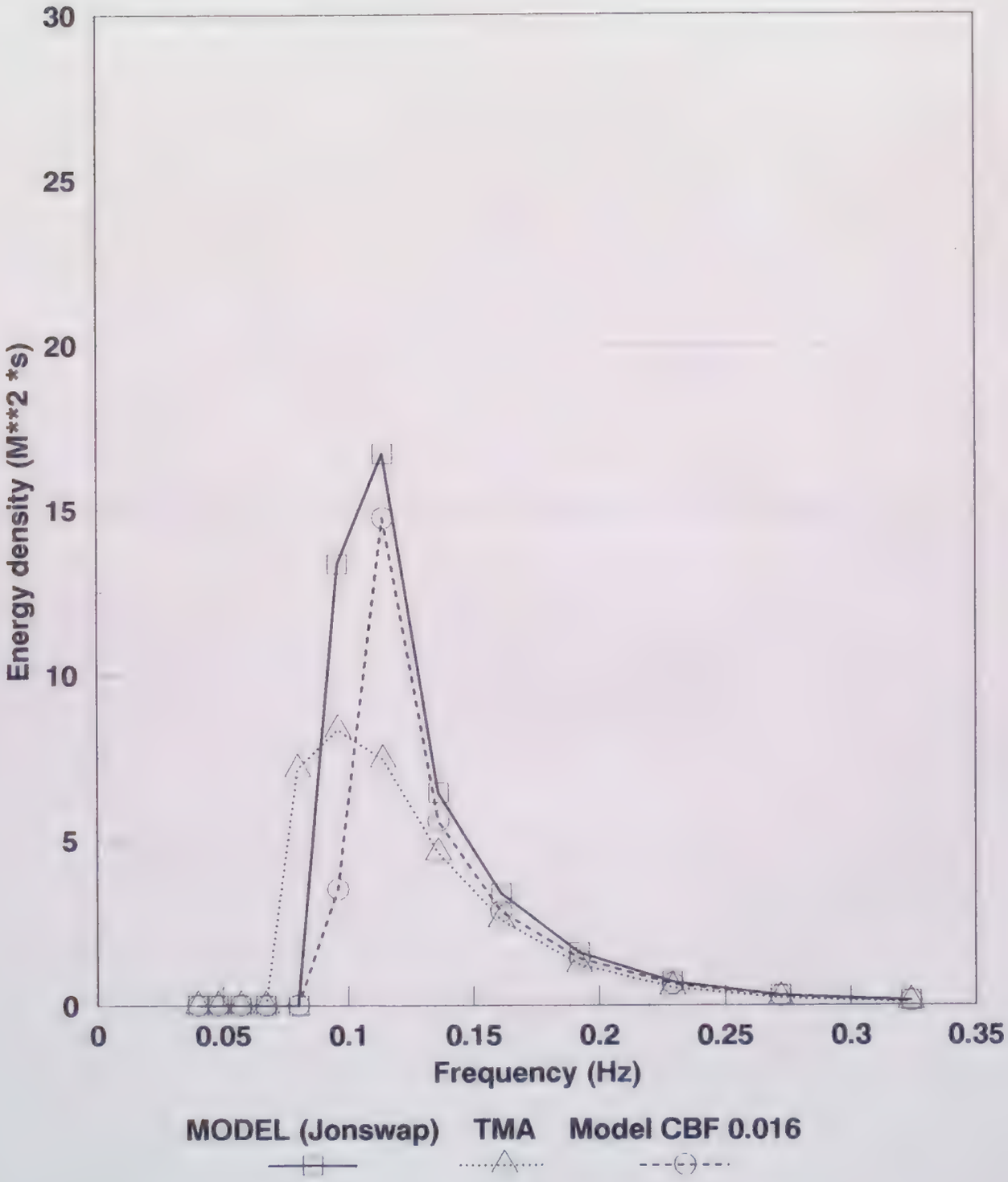


Figure 4c Shallow water spectra
DEPTH 20m U=15m/s

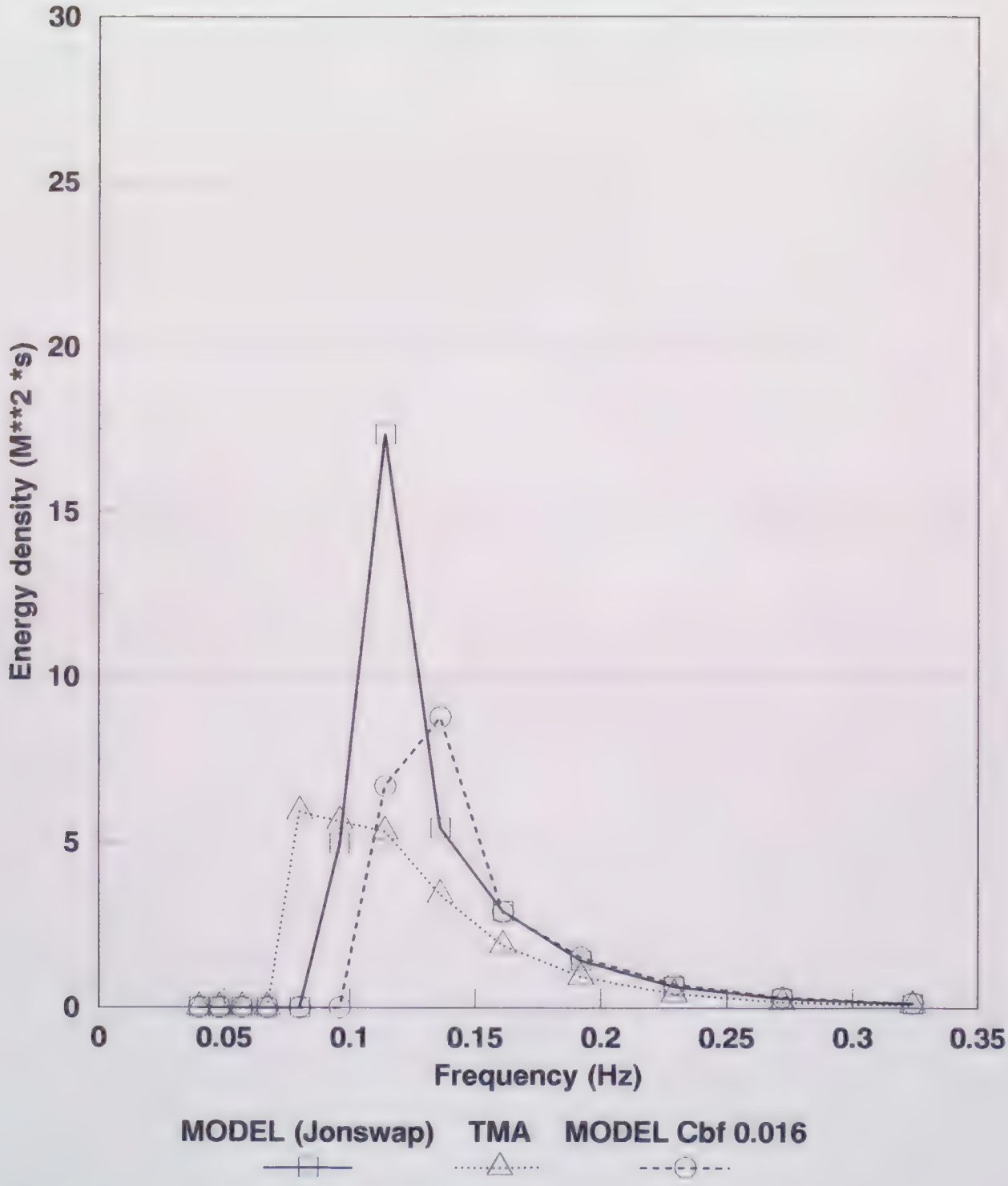


Figure 5a

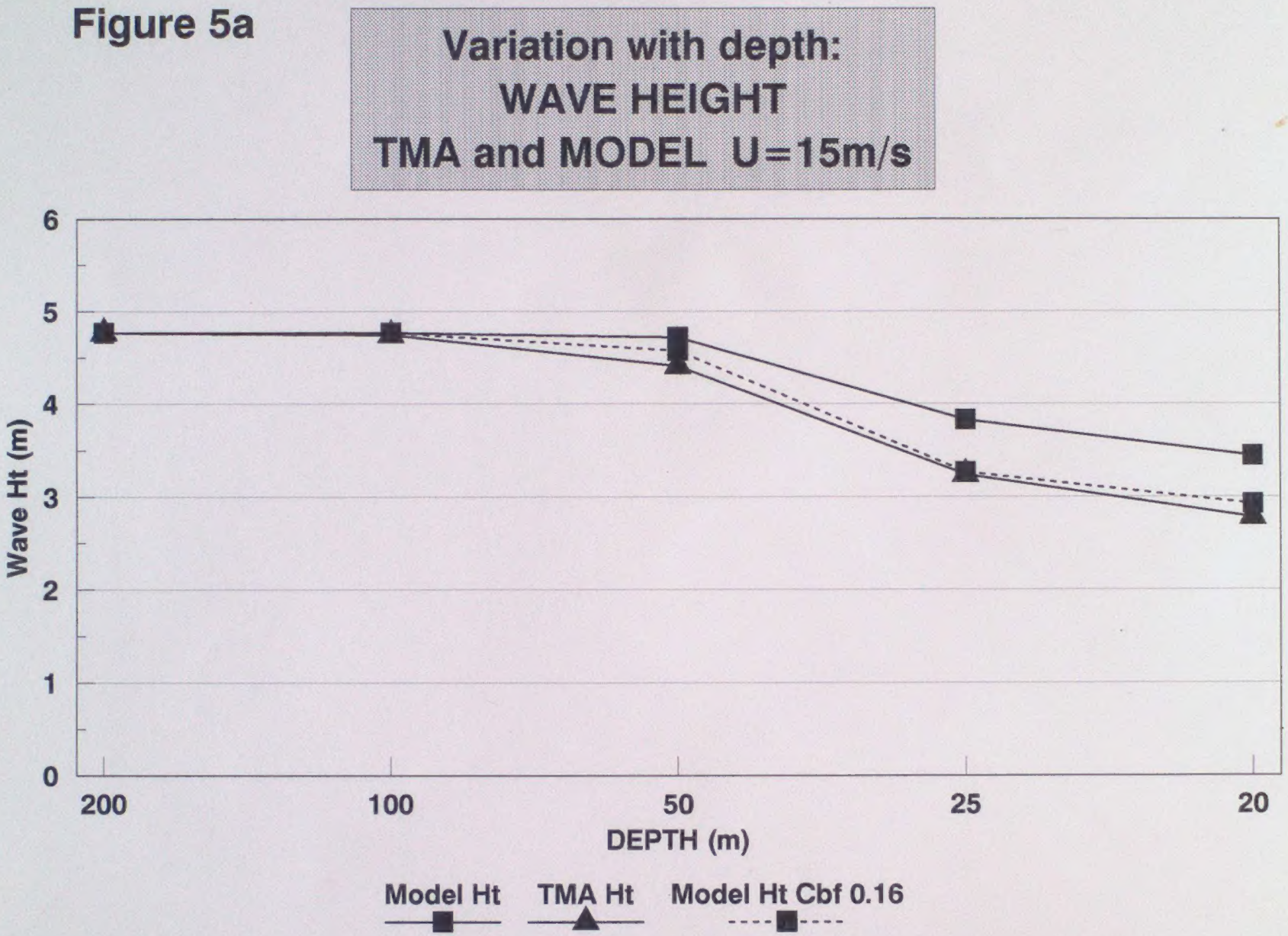

Variation with depth:
WAVE HEIGHT
TMA and MODEL U=15m/s
Wave Ht (m)
0
1
2
3
4
5
6
200
100
50
25
20
DEPTH (m)
Model Ht
TMA Ht
Model Ht Cbf 0.16

Figure 5b

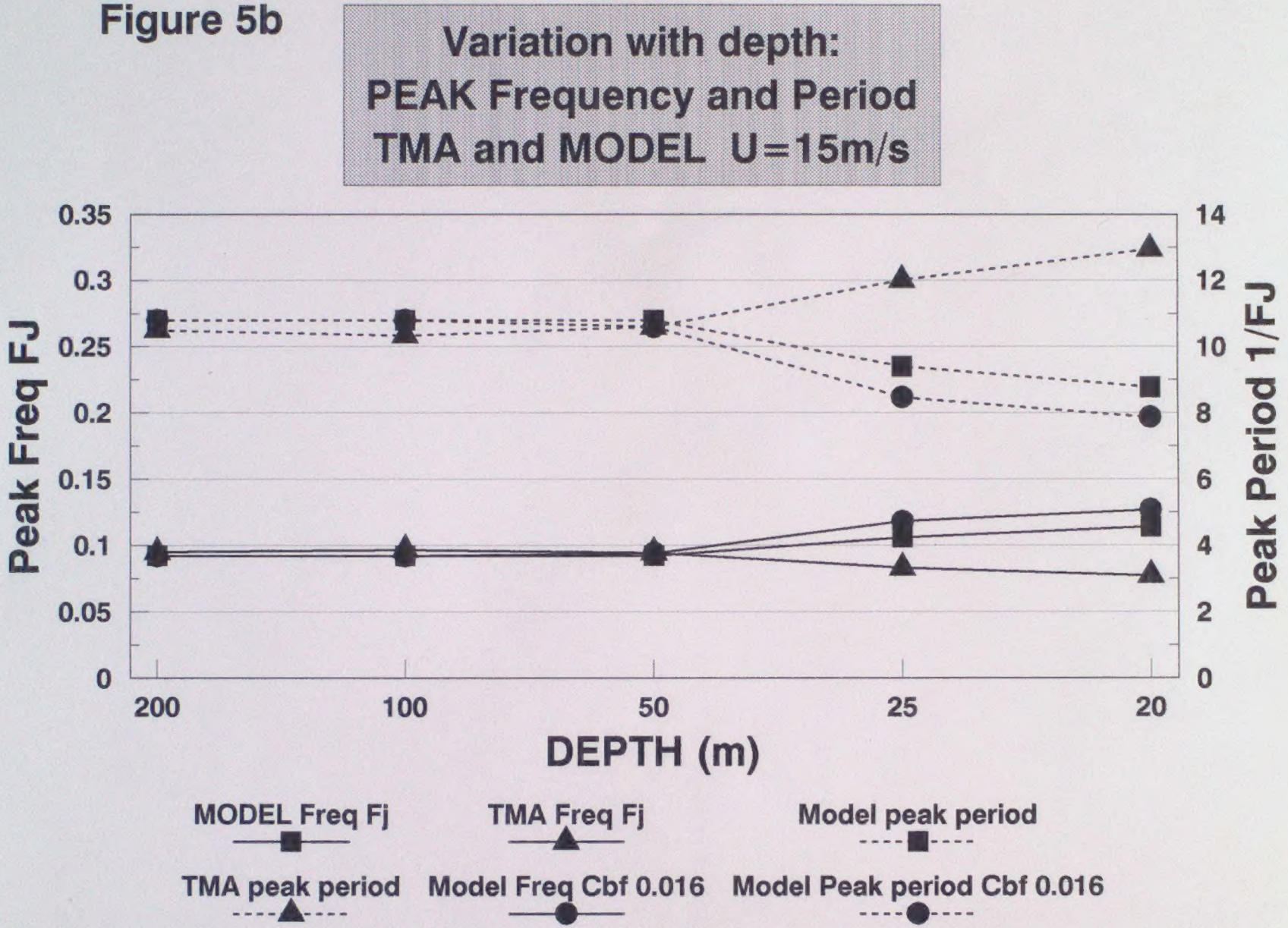

Variation with depth:
PEAK Frequency and Period
TMA and MODEL U=15m/s
Peak Freq FJ
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Peak Period 1/FJ
14
12
10
8
6
4
2
0
200
100
50
25
20
DEPTH (m)
MODEL Freq Fj
TMA Freq Fj
Model peak period
TMA peak period
Model Freq Cbf 0.016
Model Peak period Cbf 0.016

Figure 6

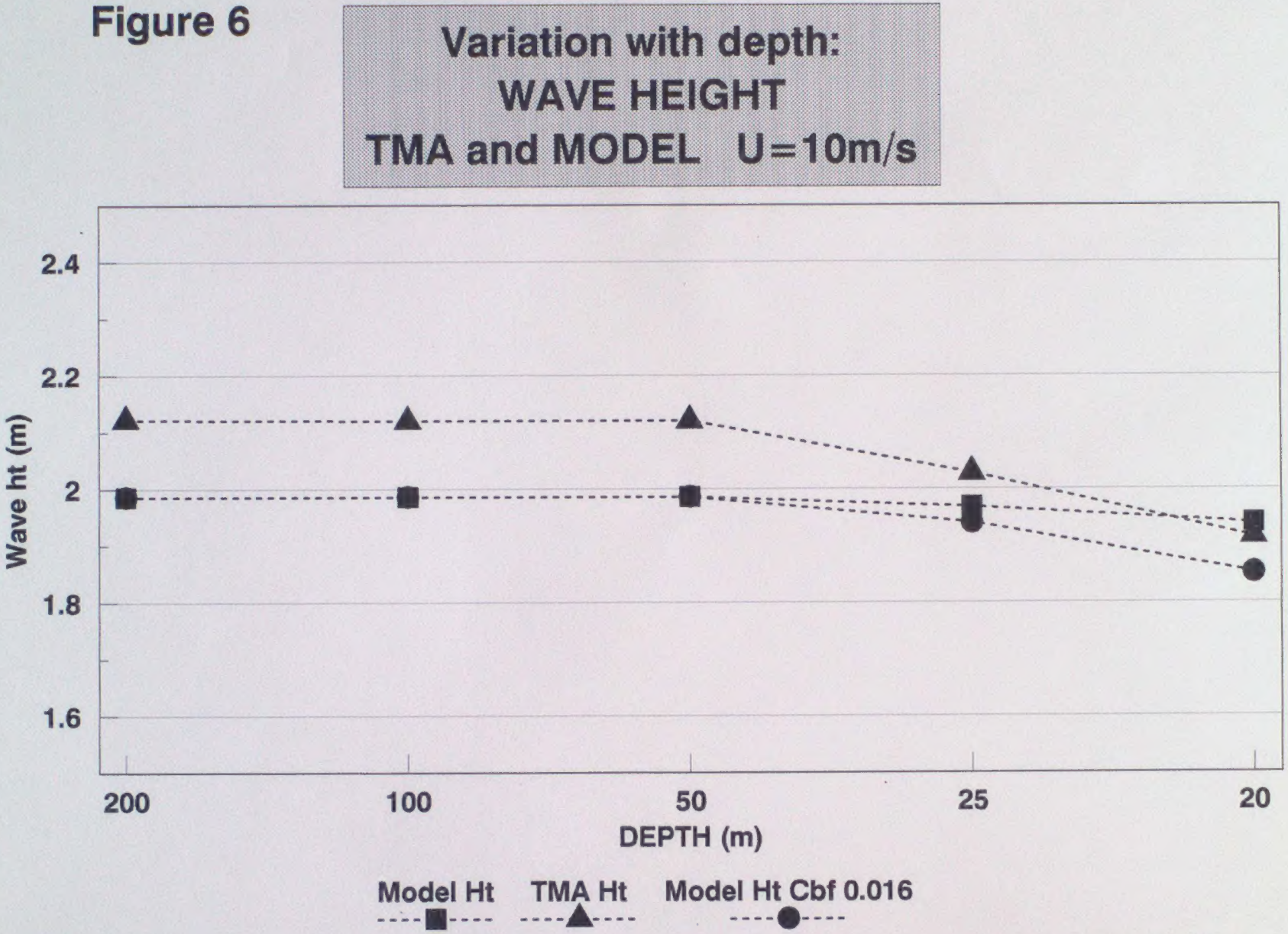
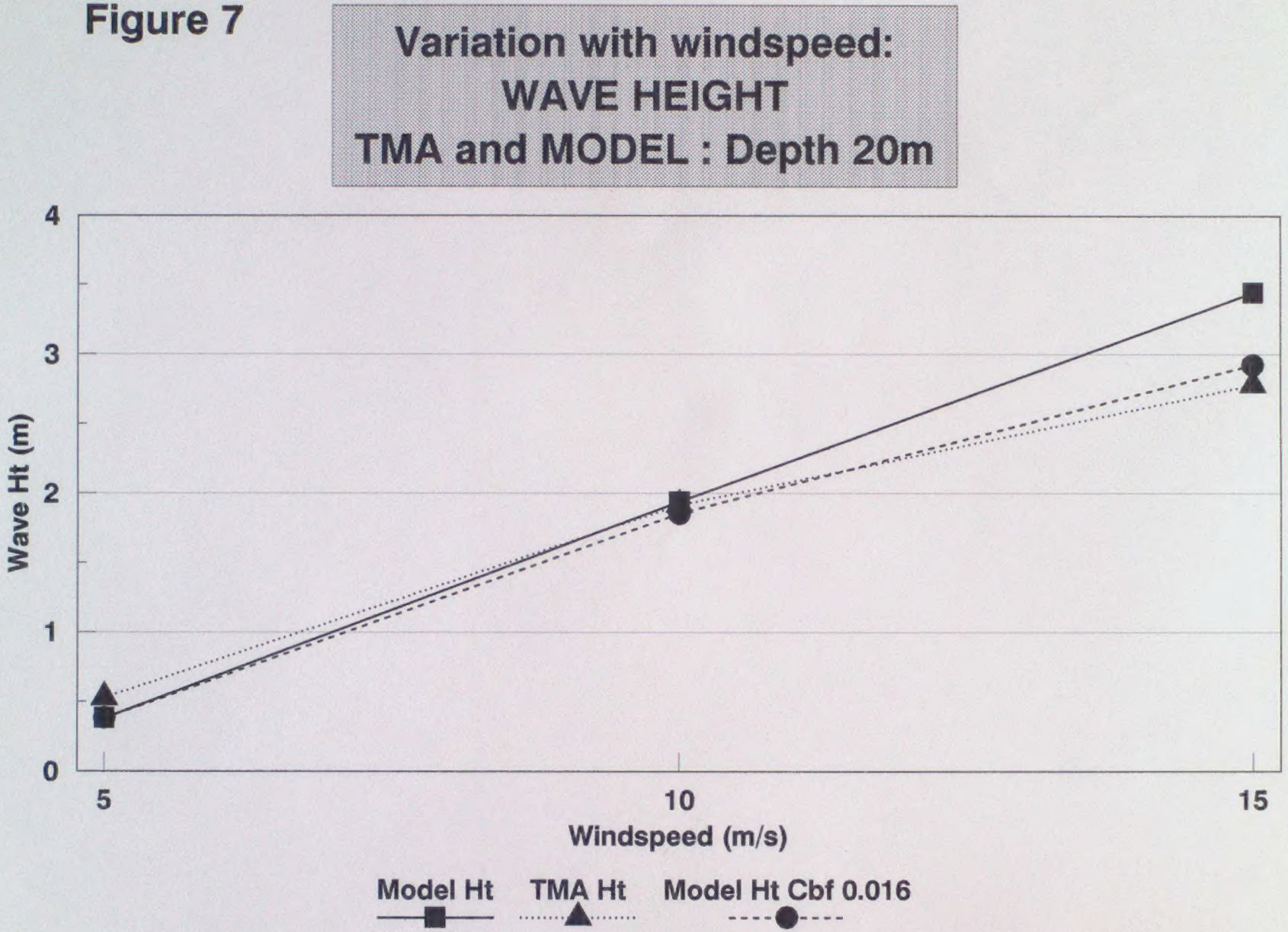

Variation with depth:
WAVE HEIGHT
TMA and MODEL U=10m/s
Wave ht (m)
2.4
2.2
2
1.8
1.6
200
100
50
25
20
DEPTH (m)
Model Ht
TMA Ht
Model Ht Cbf 0.016

Figure 7


Variation with windspeed:
WAVE HEIGHT
TMA and MODEL : Depth 20m
Wave Ht (m)
0
1
2
3
4
5
10
15
Windspeed (m/s)
Model Ht
TMA Ht
Model Ht Cbf 0.016